AN IBM-701 COMPUTER ANALYSIS OF A RESERVOIR BY MEANS OF THE MUSKAT EQUATION

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ABSTRACT

A method is presented for the expeditious analysis of performance of a solution gas drive reservoir. This rapid analysis is accomplished by the repeated solution of the Muskat equation on the IBM-701 computer. In each solution, one or more reservoir parameters are varied to determine the effect on the problem results. A small California reservoir is used to demonstrate the procedure. Graphs, illustrating the results for each parameter range, are presented.

For the sample reservoir only production, depth-pressure, and electric log data were available. Average values of PVT, viscosity, and permeability were developed from published correlation in California Oils, and entered into the Muskat prediction computer program.[2] Fifty predictions were run, and, of these, twenty-three solutions are graphically illustrated. From the graphs, a set of parameters was selected which best define the sample reservoir.

The results of this analysis indicate that more than one prediction is necessary to establish the reservoir parameters. By continuously observing the predictions, the reservoir engineer is able to decide when a reasonable approximation of the reservoir's parameters has been made. He must know, from experience and his knowledge of reservoirs, which set of parameters best defines the reservoir.

INTRODUCTION

For the last decade, Muskat's equation[1] for the depletion history of a solution gas-drive reservoir has been a reliable tool of the reservoir engineer. It undoubtedly would be used more extensively, but for the vast amount of time and labor required to complete one Muskat reservoir prediction. (An estimated sixteen man hours per prediction). Often, some of the data which the reservoir engineer must use are doubtful. It is desirable, therefore, that the Muskat prediction be run a number of times, using an array of values for the doubtful data. However, the man hours required for such a procedure would soon become prohibitive. The obvious solution to this dilemma is the high speed digital computer. Muskat's material balance equation, expressed in differential form, is ideally suited to digital computer solution.

The purpose of this paper is to illustrate how a series of Muskat predictions, obtained in a relatively short time, provided the reservoir engineer with a production analysis of his reservoir.

A small California reservoir was used to demonstrate this procedure. Data for the reservoir were provided by the Standard Oil Company of California. These data included electric logs, production records, and a limited amount of depth pressure data. No PVT data or core data were available. Porosity and water saturation data for a similar zone, in a nearby reservoir, were used to make initial estimates.

THEORY

The IBM-701 computer solution to the Muskat Prediction, once programmed, may be run, with slight alteration to the variables, until the prediction data and the production data coincide. When this occurs, it may be assumed that the prediction, extended beyond the current produced life of the reservoir, will be an accurate estimate of the reservoir's future, provided the operating procedure of the reservoir is unchanged. The computer makes a single prediction in about three minutes. However, if three predictions are run in tandem, with the same imput quantity varied in each (e. g. a different size gas cap in each run), the three predictions will be completed in less than seven minutes.

The IBM-701 program herein utilized was developed by J. E. Warren and T. D. Mueller[2] of the California Research Corporation at La Habra, California. The program includes the equations listed below:

For production above the bubble point pressure:

1. $$N_p = 7758\, \phi\, S_{oi} \left\{ \frac{1}{B_{oi}} - \frac{1}{B_o} \left[1 + M_i \left(1 - \frac{\gamma_i}{\gamma} \right) \right] \right\}$$

2. $$G_p = \frac{R_s N_p}{1000}$$

For production below bubble point pressure:

3. $$\frac{dS_o}{dp} = \frac{S_o Q + U(1 - S_o - S_w) + S_o Y\, {}^{k_g}/_{k_o} + M \quad B_o S_{oi}^{*} Q + U(1 - S_w - B_o S_{oi}^{*}) - S_{oi}^{*}\dfrac{dB_o}{dp}}{1 + \left({}^{\mu_o}/_{\mu_g}\right)\left({}^{k_g}/_{k_o}\right)}$$

with $$Q = \frac{1}{5.615\, \gamma B_o} \quad \frac{dR_s}{dp}$$

$$U = \frac{1}{\gamma} \quad \frac{d\gamma}{dp}$$

$$X = \gamma B_o \frac{\mu_o}{\mu_g}$$

$$Y = \frac{X}{\gamma B_o^2} \frac{dB_o}{dp}$$

4. $R = R_s + 5.615 X \, {}^{k_g}/_{k_o}$

5. $N_p = 7758 \, \phi \left(\frac{S_{oi}}{B_{oi}} - \frac{S_o}{B_o} \right)$

6. $$G_p = 43.56 \, \phi \left\{ \left[\frac{R_s S_{oi}}{5.615 B_{oi}} - \frac{R S_o}{5.615 B_o} + \gamma_i S_{gi} - \gamma S_g \right] + M \left[\gamma_i \left(1 - \frac{S_{oi}^*}{B_{oi}} - S_w \right) - \gamma \left(1 - \frac{S_{oi}^*}{B_o} - S_w \right) + \frac{R_{si} S_{oi}}{5.615 B_{oi}} - \frac{R_s S_{oi}^*}{5.615 B_o} \right] \right\}$$

where:

N_p = Cumulative oil production in bbl of STO/acre ft.

G_p = Cumulative gas production in MCF/acre ft.

R = Produced gas-oil ratio in SCF/STbbl.

All other symbols are defined in the appendix, page i.

The computer program is designed to solve these equations, and to record their solution as illustrated by Figure 1.

To adapt the computer program to a particular reservoir, the PVT data, viscosity data, and permeability data for that reservoir are introduced into the program. To consider a new reservoir, one simply substitutes new data.

RESERVOIR HISTORY

From the electric logs provided, a sand count was made of the producing sand, and a contour map was developed. (see Figure 2). It was

RESERVOIR B

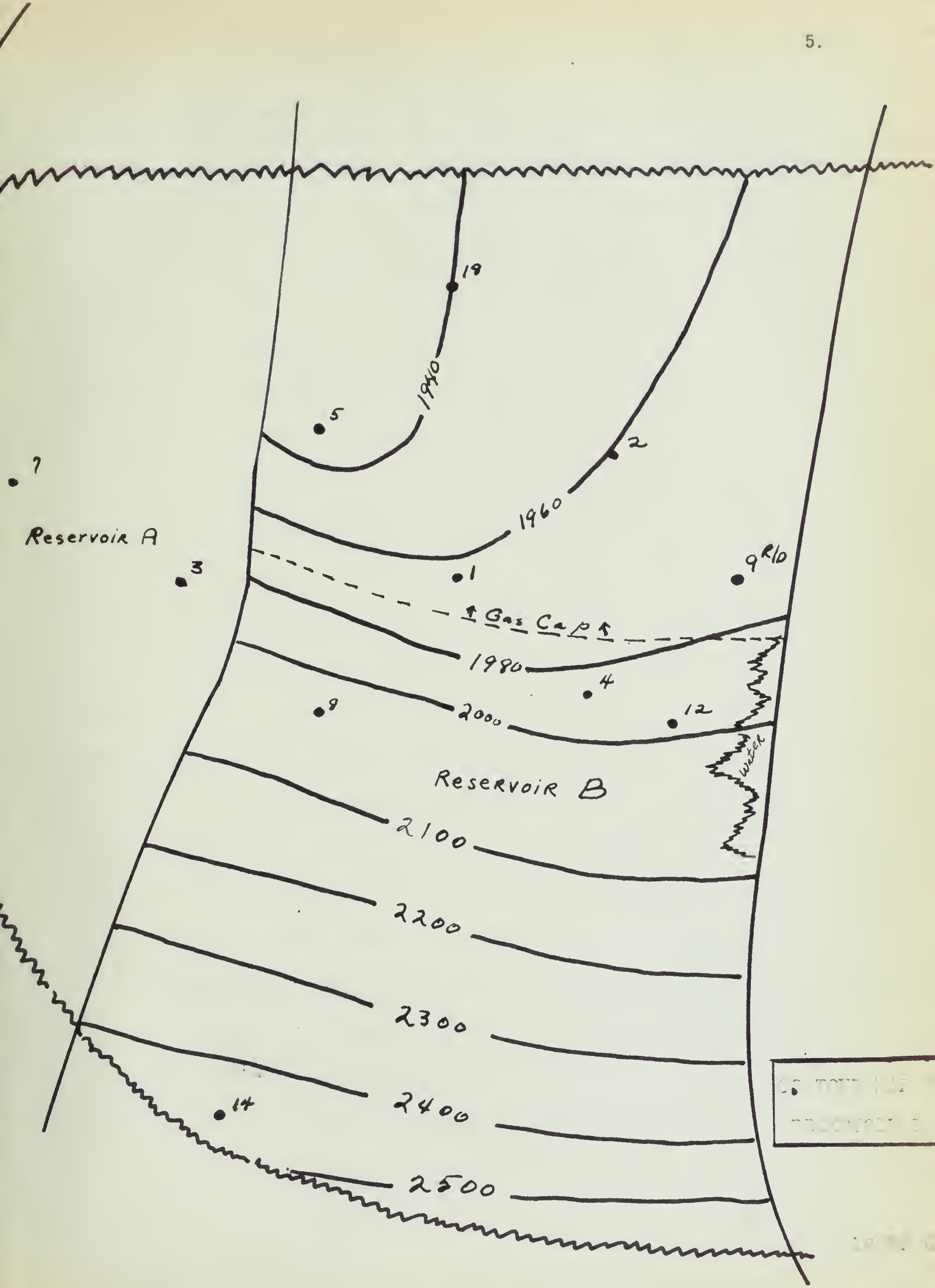
IBM 701 COMPUTER SOLUTION OF MUSKAT PREDICTION
DATE BP-1200

CASE 1 NORMAL DEPLETION GAS CAP
POROSITY .25 WATER .30 GAS CAP .5 OIL SAT.15

PRESSURE PSIA .XXXXX	OIL RECOVERY PER CENT .XXXXXXX	SATURATION PERCNT OIL .XXXXXXX PERCNT GAS .XXXXXXX	ACREFOOT RECOVERY OIL BBLS .X XXXX GAS MF .XXXXXX	GAS-OIL RATIO CFB PRODUCING .XXXX CUMULATIVE .XXXX	CUM GAS INJECTED 0000000000 MCF ACREFT .XXX
0000000000	0000000000				
0150000000	0000000000	0700000000	0000000000	0000000000	0000000000
0000000000	0000000000	0000000000	0000000000	0000000000	0000000000
0140000000	0041837534	0663373344	0047935374	0006869420	0000000000
0000000000	0000000000	0036626655	0033327381	0006952565	0000000000
0130000000	0074219347	0634032794	0085036851	0008908629	0000000000
0000000000	0000000000	0065967205	0074522671	0008763573	0000000000
0120000000	0103186192	0607655183	0118225627	0011646660	0000000000
0000000000	0000000000	0092344816	0120150128	0010162781	0000000000
0110000000	0127783612	0584789805	0146408132	0015872915	0000000000
0000000000	0000000000	0115210194	0169775154	0011596019	0000000000
0100000000	0148345055	0565108516	0169966415	0019607777	0000000000
0000000000	0000000000	0134891483	0222580504	0013095557	0000000000
0090000000	0166248130	0547611393	0190478872	0024433709	0000000000
0000000000	0000000000	0152388606	0277641199	0014575957	0000000000
0080000000	0181595675	0532167819	0208063329	0029851558	0000000000
0000000000	0000000000	0167832180	0334495589	0016076623	0000000000
0070000000	0194946477	0518359356	0223360016	0035136400	0000000000
0000000000	0000000000	0181640643	0392664784	0017579904	0000000000
0060000000	0206709378	0505879648	0236837364	0041051325	0000000000
0000000000	0000000000	0194120351	0451785579	0019075772	0000000000
0050000000	0217419080	0494355046	0249108011	0045937477	0000000000
0000000000	0000000000	0205644953	0511549003	0020535228	0000000000
0040000000	0227440746	0483532019	0260590339	0048806263	0000000000
0000000000	0000000000	0216467980	0571730158	0021939806	0000000000
0030000000	0237436136	0472982649	0272042562	0048912705	0000000000
0000000000	0000000000	0227017350	0632126843	0023236321	0000000000

COMPUTER RESULTS

FIGURE 1


19
1940
5
2
7
1960
Reservoir A
3
1
9 R/10
Gas Cap
1980
4
8
2000
12
Water
Reservoir B
2100
2200
2300
2400
14
2500

apparent, from the production data and from numerous cross sections of the producing zone and the two zones above it, that the reservoir provided was actually three reservoirs, separated from one another by fault zones. It was decided that only one of these reservoirs, Reservoir B, (illustrated in Figure 2) would be used to demonstrate the IBM-701 prediction procedure.

The contour map of Reservoir B shows that the reservoir contains nine wells. Of the nine wells drilled, only two are currently producing. Well #5 was the first well brought in (November 1949) with an initial production of 400 bbls/D. It produced until December 1950 (a total of 82,000 bbls), when it was shut in with a maximum gas-oil ratio of 7500. Well #18 was completed in January 1950, initial production 9 bbls/D. It produced until December, 1951 when its maximum gas-oil ratio was 6700. It produced a total of 2,000 bbls of oil. Well #1 was drilled in June 1950, and came in a gas well. It was immediately shut-in. Well #8 was completed in November 1950; its initial production was 380 bbls/D. It is currently producing 31 bbls/D. Its gas-oil ratio has climbed from 280 to 4,000. It has produced a total of 185,000 bbls. of oil. Well #9 was drilled in January 1953. It came in a gas well and was shut in after producing 3,600 MCF of gas and 33 bbls. of oil. Well #12 was completed in January, 1956 with an initial production of 49 bbls/D. It has produced 22,800 bbls. to date and is currently producing 51 bbls/D. This well initially produced 118 bbls/D of water, and is now producing 272 bbls/D. It is the only well in the field that has produced a sizable amount of water. Wells #2, #4, and #14 were dry holes. As illustrated in Figure 2, the entire

northern part of Reservoir B, which includes wells #5, #18, #1, #2, and #9 R/D, is a gas cap. There is also evidence of water influx in the eastern edge of the reservoir near well #12.

The total oil produced to date from Reservoir B (July 1957) is 507,356 bbls. of stock tank oil. This oil has an average gravity of 35° API. 673,457 MCF of gas has been produced.

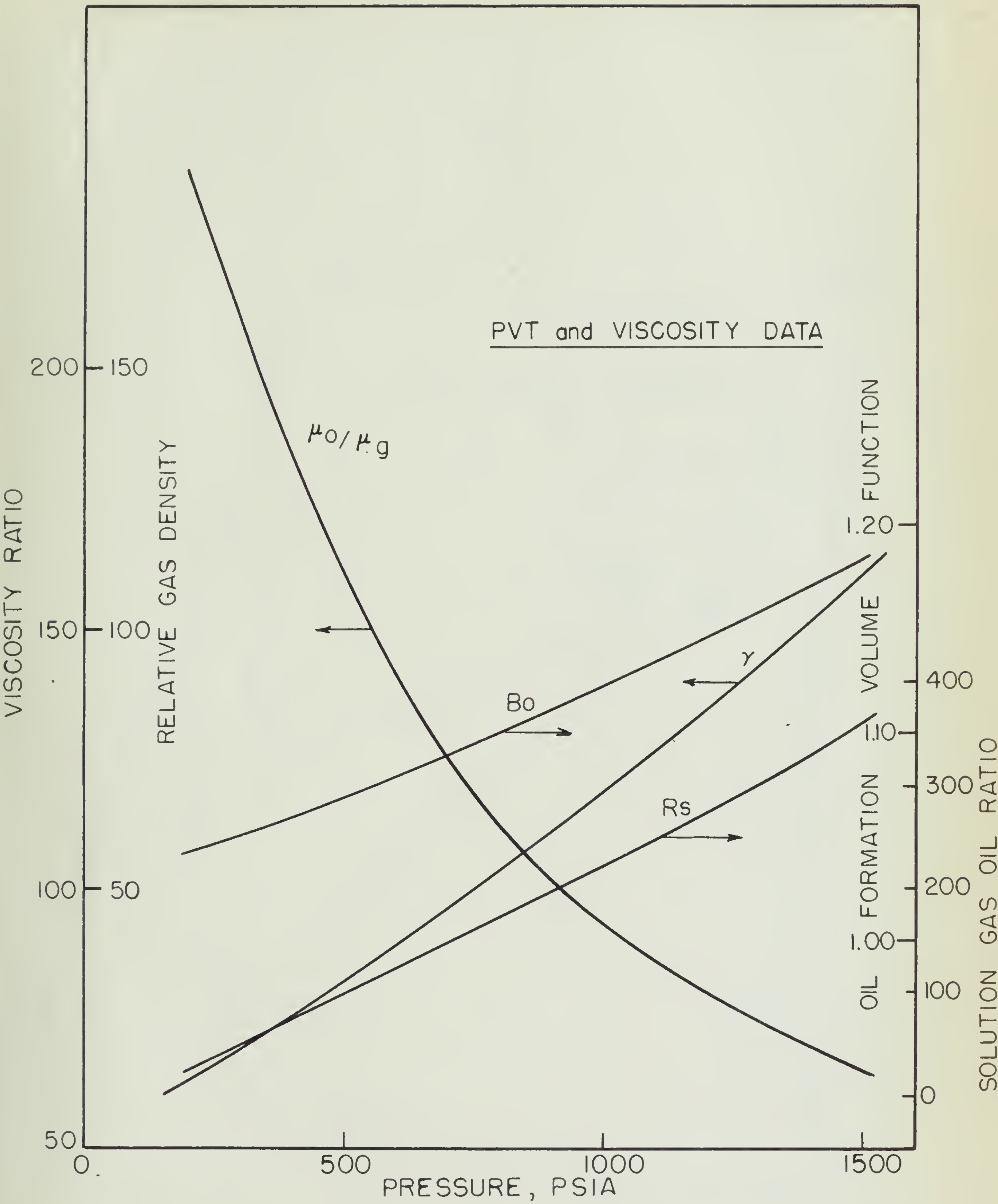
PREDICTION PROCEDURE

Production data provided an average API gravity of 35° for stock tank oil, an average gas gravity of 0.73 for the reservoir gas at standard stock tank conditions, and an average reservoir temperature of 135° F. Standing's[3] correlation charts were used to obtain PVT data, and Beal's[4] correlations were used for the viscosity data. PVT data and viscosity data, as a function of pressure, are included as Graph A.

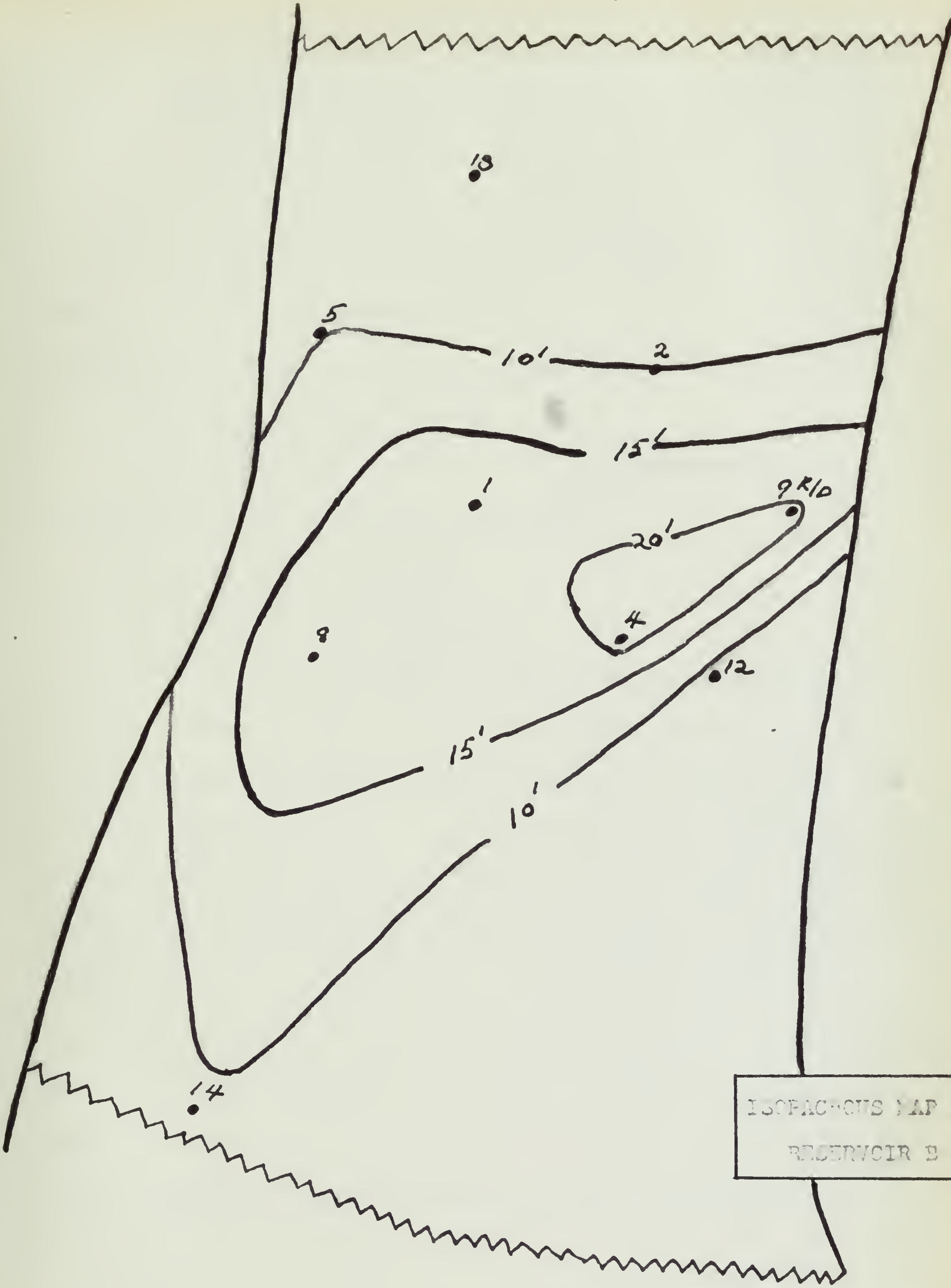
From the sand count of Reservoir B, an isopachous map was constructed (Figure 3). A volumetric estimate of sand in place was computed to be 2,520 acre feet. Porosity and connate water saturation were estimated to be 0.35 and 0.45 respectively. These were the average values of twenty-two porosities and water saturations taken from sand in an adjacent reservoir. The adjacent sand was considered to be similar. Using the equation:

$$N = \frac{(7756)\,(\text{acre ft.})\,(\phi)\,(1 - S_w)}{B_{oi}},$$

an initial estimate of oil in place was computed to be 2.88 million stock tank barrels. (Sample calculation, page ii in appendix illustrates this procedure).


PVT and VISCOSITY DATA
VISCOSITY RATIO
RELATIVE GAS DENSITY
OIL FORMATION VOLUME FUNCTION
SOLUTION GAS OIL RATIO
PRESSURE, PSIA
μ_o/μ_g
Bo
γ
Rs
200 150
150 100
100 50
50
0
500
1000
1500
1.20
1.10
1.00
400
300
200
100
0

GRAPH A



ISOPACHOUS MAP OF
RESERVOIR B

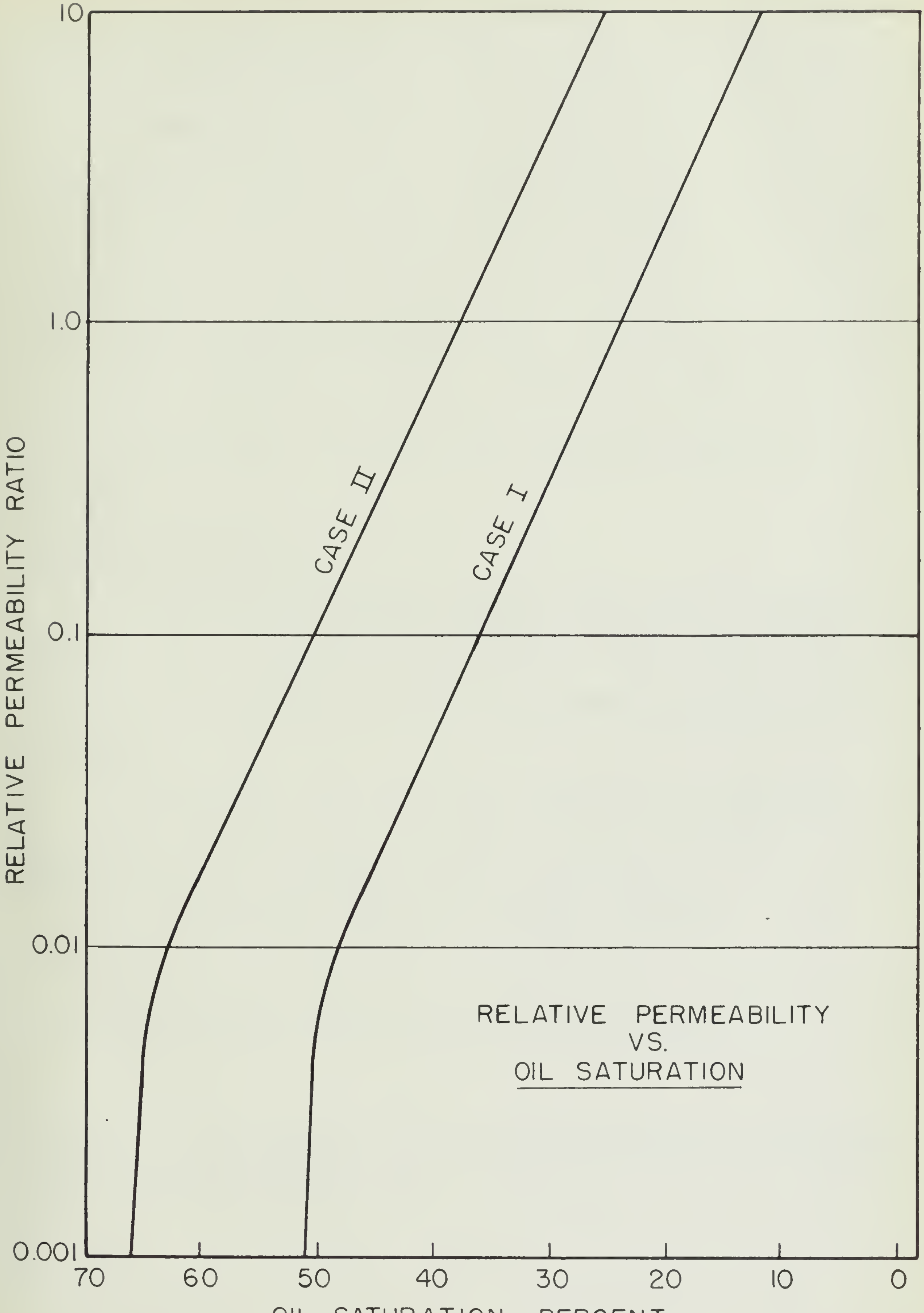
This estimate was substantiated by the Schilthuis Material Balance Equation when an allowance was made for gas production from the gas cap. A second calculation of N was made using new values of porosity and connate water saturation, ϕ = 0. 25 and S_w = 0. 30. These estimates were taken as average for California sandstone. The initial estimates are on the extreme end of the range for California sandstones. The new values of porosity and water saturation give a value of N equal to 3.14 million barrels of stock tank oil. The change in N is comparatively slight.

A curve of oil saturation vs. relative permeability, (Graph B), was developed by solving the equations listed below:

7. $S_o = (1 - S_w)(1 - \frac{\Delta N}{N}) \frac{B_o}{B_{oi}}$

8. $k_g/k_o = (R - R_s)\, \mu_g/\mu_o \frac{1}{\gamma B_o}$

Static bottom hole pressure, cumulative oil and gas production, and instantaneous gas-oil ratios were plotted vs. time in years. (see Graphs C & D). From the correlations established, R and Δ N (bbls. of STO) were taken as functions of pressure. Dividing Δ N by N gave Δ N/N as a function of pressure. Since two N's were computed, two cases, Case I, where ϕ = 0. 35 and S_w = 0. 45 and Case II, where ϕ = 0. 25 and S_w = 0. 30 were developed. PVT and viscosity data, as functions of pressure, were taken from Graph A. Equations (7) and (8) were solved as functions of pressure in decrements of 100 psi from 1500 to 600 psia. From the correlation of relative permeability and oil saturation for the same decrement of pressure, two curves of K_g/K_o vs. S_o for Case I and Case II. Both curves were extended into the region of



10
1.0
0.1
0.01
0.001
RELATIVE PERMEABILITY RATIO
CASE II
CASE I
RELATIVE PERMEABILITY
VS.
OIL SATURATION
70
60
50
40
30
20
10
0
OIL SATURATION, PERCENT

GRAPH B

higher gas saturation by continuing them parallel to the curve developed by Arps[5] for the average value of relative permeability in sandstones. Once the PVT, viscosity, and relative permeability data were compiled, they were included in the computer program. Experience with the computer program indicates that most functional data, used by the computer, must be mathematically smoothed. The smoothing of the PVT and viscosity data was accomplished by third order polynomial equations. A seventh order equation is usually required for K_g/K_o functions. However, in this particular case, the curve from Arp's average was sufficiently smooth to enter the program directly.

PREDICTION RESULTS

It was anticipated that the predicted results could be brought to superimpose upon the observed production data. This was to be accomplished by a system of altering the problem variable until the desired results were reached.

For the first prediction, it was decided to investigate the Case I parameters. The following initial conditions were set:

P_i = 1500 psia, bubble point pressure = 1500 psia

$S_w = 0.45$, $\phi = 0.35$, $m = 0.1$, $S_{oi}^* = 0.15$

The predicted results are illustrated in Figures 4.0 and 4.1. *

*Predicted data was illustrated in all figures by a solid curve. Observed data was shown as circled points.

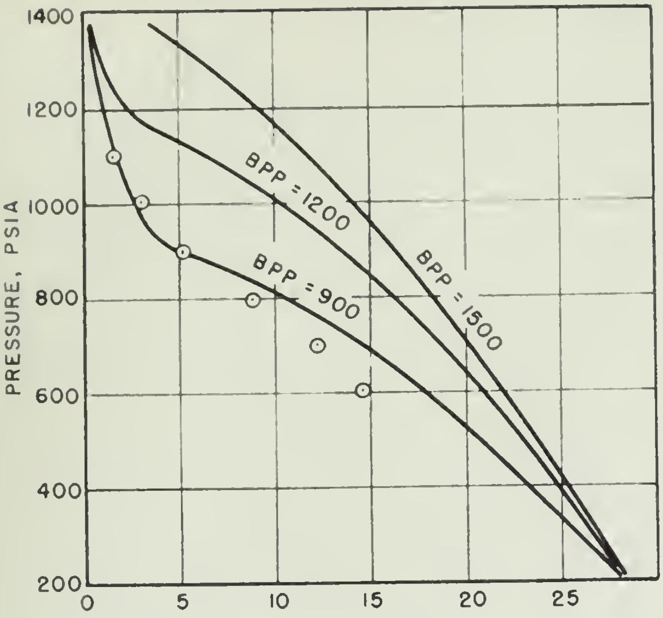


FIG. 5.0

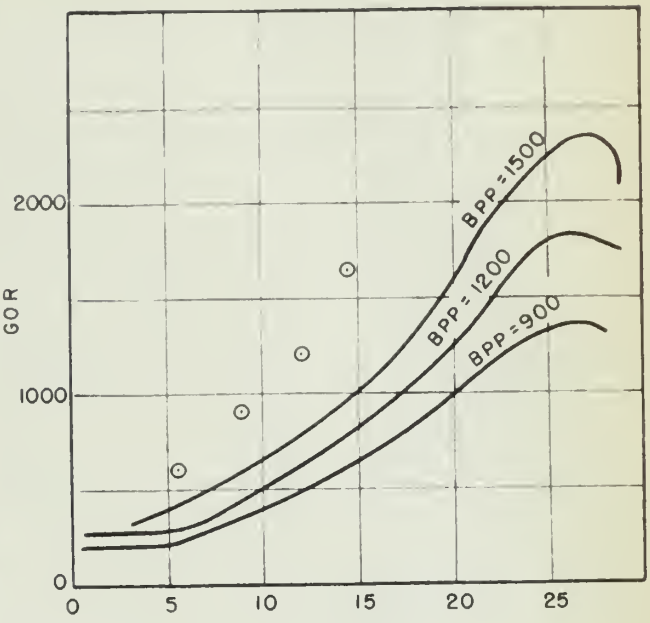


FIG. 5.1

Figures 5.0 and 5.1 show that lowering the bubble point causes the predicted data to begin to approach the observed data. The best approximation of % $\Delta N/N$ is for a BPP = 900 psia, but the declining slope after P = 900 is too gentle.

After this prediction was made, the effect of varying porosity, water saturation, viscosity, gas cap and initial pressure was investigated. None of these variations produced a unique solution. Table 1.0 lists the parameters investigated.

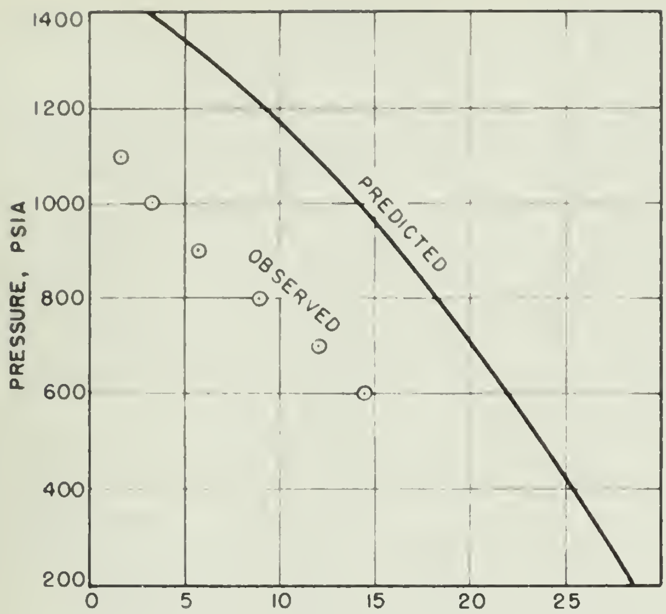


FIG. 4.0

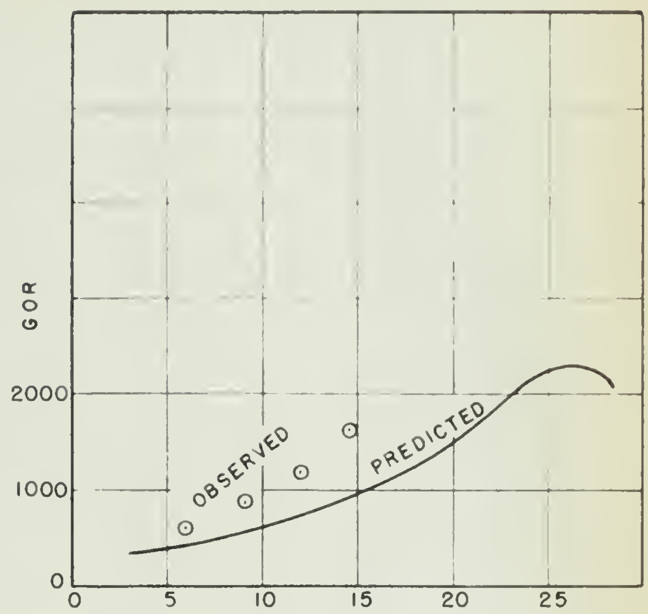


FIG 4 1

Very little agreement is shown between observed and predicted data.

To investigate the effect of bubble point on the prediction a second run was made. This was a tandem run with a different bubble point used for each prediction. The following initial conditions were set:

P_i = 1500 psia, BPP = 1500, 1200, 900 psia, S_w = 0.45, ϕ = 0.35, m = 0.1, S_{oi} = 0.15

The predicted results are illustrated in Figures 5.0 and 5.1.

TABLE 1.0 CASE I Parameters Not Illustrated on Graphs

Initial Pressure	1500	1500	1500	1500	1500	1500 1400 1300	1500	1300
Bubble Point Pressure	1500	900	900	900	900	1500	900	1100 1000 900
Porosity	0.35	0.35	0.35	0.30 0.35 0.40	0.35	0.35	0.35	0.35
Connate Water Saturation	0.45	0.45	0.45	0.45	0.40 0.45 0.50	0.45	0.45	0.45
Gas Cap Vol. / Oil Vol. (m)	0.1 0.5 0.9	0.1 0.5 0.9	0.1	0.1	0.1	0.1	0.1	0.1
M Viscosity Ratio	1.0	1.0	0.8 M 1.0 M 1.5 M	1.0	1.0	1.0	1.0	1.0
S_{oi}*	0.15	0.15	0.15	0.15	0.15	0.15	0.20 0.15 0.10	0.15
% $\Delta N/N$	28.6 33.1 36.2	28.1 33.6 37.2	30.4 28.1 24.2	28.1	26.4 28.1 30.1	28.6 27.8 26.9	28.0 28.1 28.3	28.5 28.1 25.4

The Case II parameters were next investigated. For the third prediction, the following initial conditions were set:

P_i = 1500 psia, BPP = 1500 psia, S_w = 0.30, ϕ = 0.25, m = 0.5,

S_{oi}* = 0.15

The predicted results are illustrated in Figures 6.0 & 6.1.

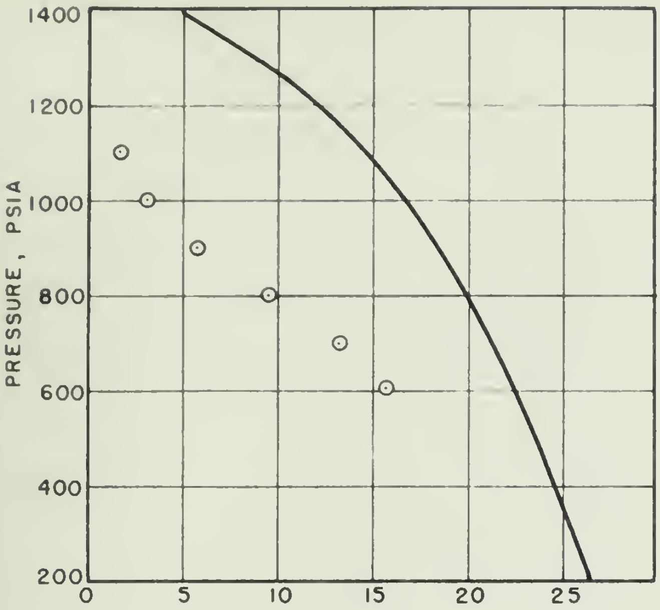


FIG. 6.0

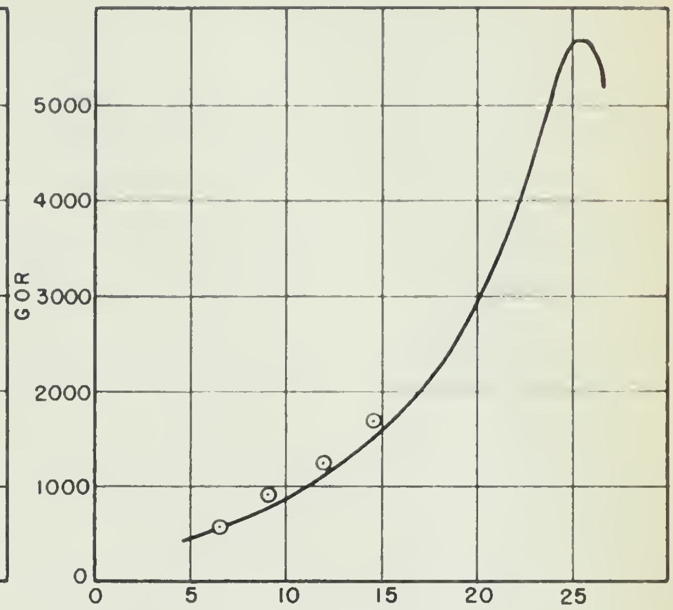


FIG. 6.1

Here the GOR curve showed slight agreement. No agreement was apparent for oil production.

The fourth prediction was used to investigate the effect of gas cap size. The following initial conditions were set:

P_i = 1500, BPP = 1500 psia, S_w = 0.30, ϕ = 0.25, m = 0.1, 0.5 & 0.9, $S_{oi}*$ = 0.15

The predicted results are illustrated in Figures 7.0 and 7.1.

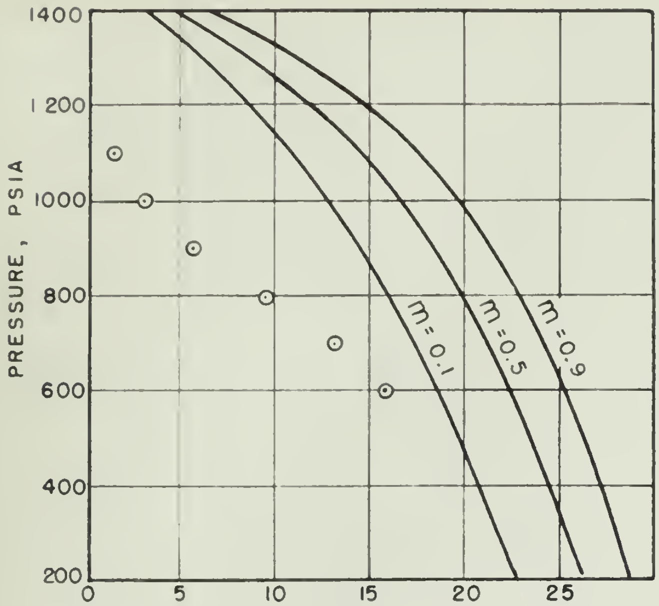


FIG. 7.0

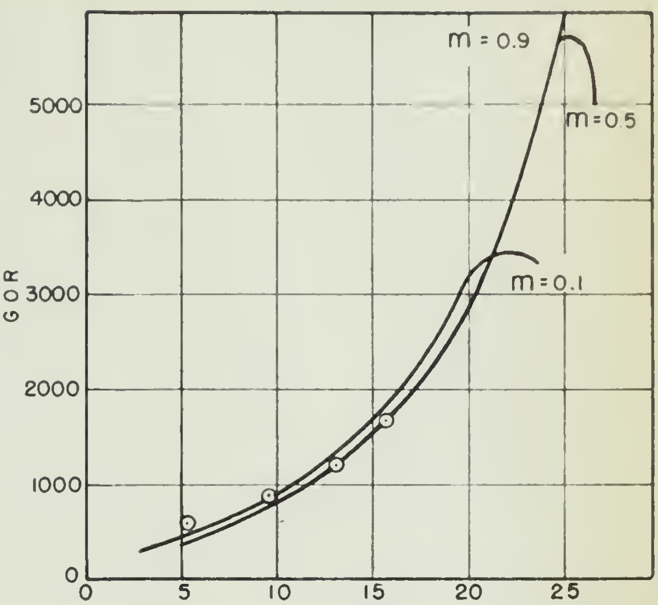


FIG. 7.1

The fourth prediction indicated that the gas cap size does have a significant effect upon gas and oil produced. It suggests that $m = 0.1$ is probably the more likely size for this reservoir. There was no agreement between predicted and observed data for oil production, but predicted and observed gas-oil ratio compared well.

The fifth prediction was used to demonstrate the effect of bubble point upon production. For this prediction the following initial conditions were set:

P_i = 1500 psia, BPP = 1500, 1200, 900 psia, S_w = 0.30, ϕ = 0.25

m = 0.1, $S_{oi}*$ = 0.15

The predicted results are illustrated in Figures 8.0 and 8.1.

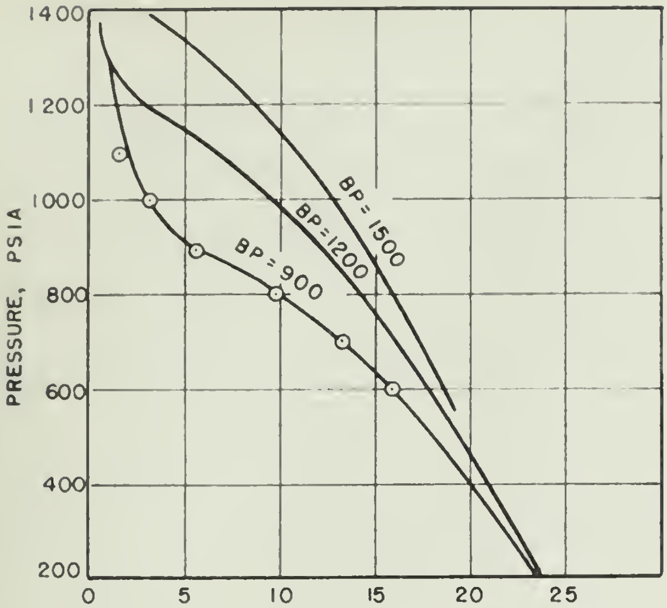


FIG. 8.0

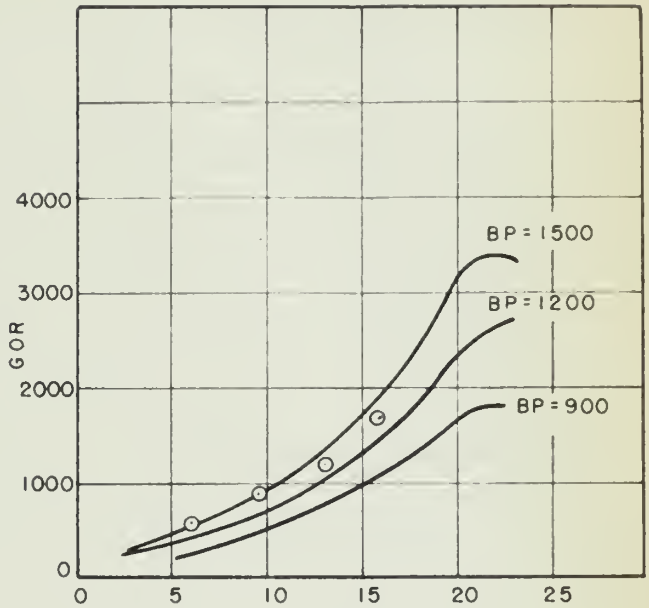


FIG. 8.1

The fifth prediction showed that lowering the bubble point pressure caused the predicted oil curve to approach the curve of the actual oil produced. At a bubble point of 900 psia, the predicted oil curve superimposed upon the actual produced oil curve. However, the corresponding GOR curve at BPP = 900 psia was too low. This may be explained by examining the history of Reservoir B. The excessively high gas oil ratios which occur at various times throughout the reservoir's history, strongly suggest that gas was produced from the gas cap. Since the Muskat equation does not account for gas production directly from the gas cap, the computer program was not designed to reflect it. It is concluded that the difference between GOR computed and actual GOR is due to the gas produced directly from the gas cap. The prediction indicates that slightly over half of the gas produced from the initial pressure of 1500 psia to 600 psia was produced directly

from the gas cap. A new estimate of actual GOR curve was made excluding the gas cap production. This new GOR will be reflected on the remaining curves. The fifth prediction suggests that a bubble point of approximately 900 psia is most likely for Reservoir B. Using this new bubble point results in a new value of N, based on a change in B_{oi}. The change is slight, however, and not sufficient to alter the K_g/K_o vs. S_o curve.

It should be noted on figure 8.0 that the recoverable oil at abandonment pressure of 200 psia was roughly the same for all three bubble points.

To substantiate the earlier decision to use a gas cap size $m = 0.1$, in the sixth prediction, a variable gas cap was once more employed; this time with the bubble point pressure at 900 psia. The following initial conditions were set:

$P_i = 1500$ psia, BPP = 900 psia, $S_w = 0.30$

$\phi = 0.25$, $m = 0.1$, 0.5, & 0.9, $S_{oi} = 0.15$

The predicted results are illustrated in Figures 9.0 and 9.1.

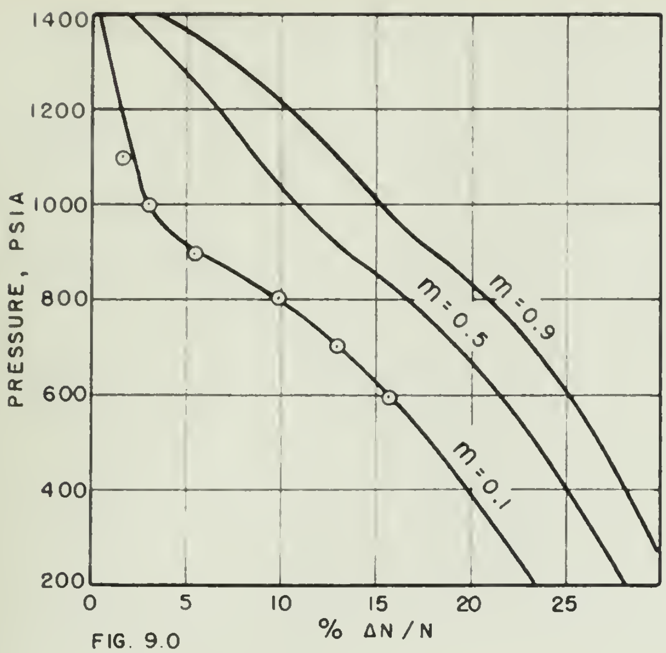


FIG. 9.0

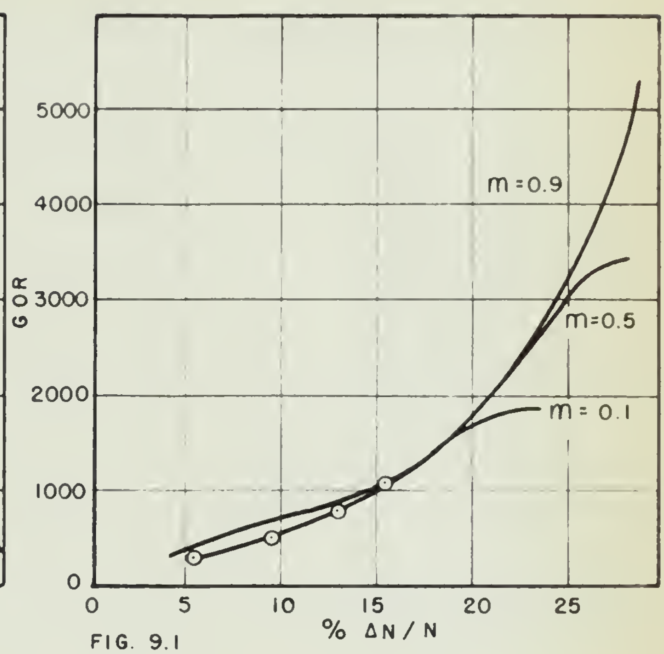


FIG. 9.1

Once again, an m = 0.1 appeared to be the best estimate for gas cap size in Reservoir B.

To investigate the influence of viscosity, the seventh prediction was made with the following initial conditions:

P_i = 1500 psia, BPP = 900 psia, S_w = 0.30, ϕ = 0.25,

m = 0.1, S_{oi}* = 0.15, and 0.8M, 1.0M & 1.5M where M = μ_o / μ_g

The predicted results are illustrated in Figures 10.0 and 10.1.

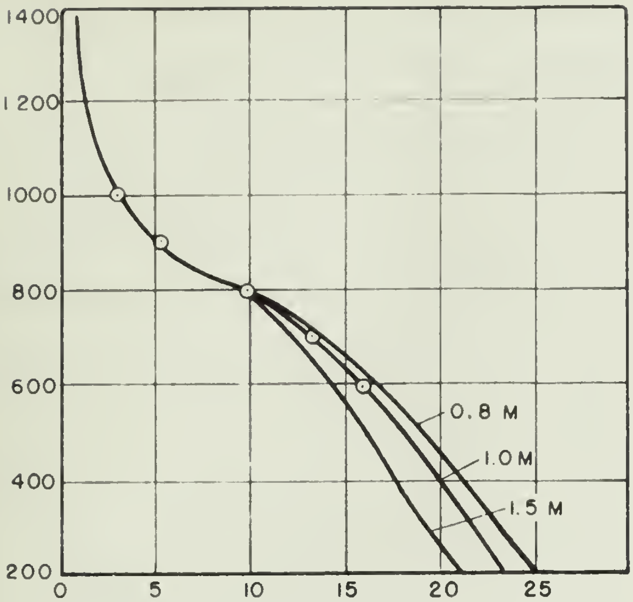


FIG. 10.0

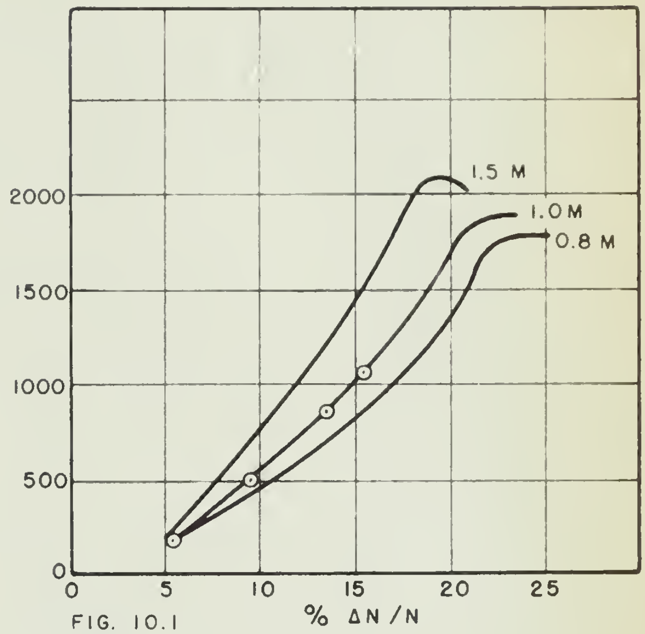


FIG. 10.1

As one would expect a higher viscosity results in less recoverable oil and a consequent higher GOR. The results indicated that the initial estimate of oil viscosity (M = μ_o/μ_g x 1. 0) is the best approximation for the actual production data.

In the eighth prediction, the influence of porosity was investigated. Changing porosity changed the value of N, the initial oil in place. It did not change the value of % Δ N/N because porosity cancels out of this ratio. However, in the calculation of oil and gas produced per acre foot, porosity is reflected. Converting the acre feet production to predicted reservoir production permits a comparison with actual production, Graph D.

The change in N for each value of porosity resulted in a very slight shift of K_g/K_o along the S_o axis (Graph B and Equation 7). The shift was

so slight that Arp's average curve was not varied.

The eighth prediction initial conditions were set as follows:

P_i = 1500 psia, BPP = 900 psia, S_w = 0.30, ϕ = 0.20, 0.25 & 0.30, m = 0.1, S_{oi}* = 0.15

The predicted results are illustrated in Figures 11.0 and 11.1.

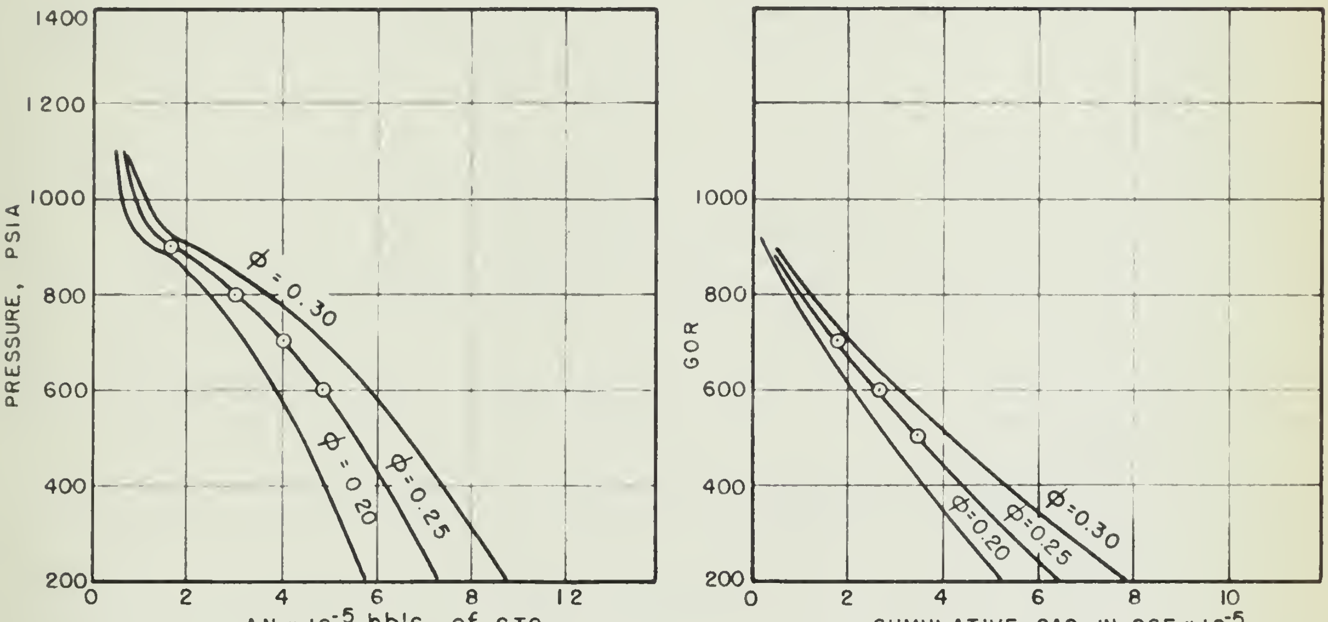


FIG. 11.0 FIG. 11.1

This prediction is compared with actual production data illustrated on Graph D. The production data is not altered by dividing it by N. It is concluded that for Reservoir B, a porosity of 0.25 is the only porosity which will give a prediction that satisfies actual production data. This condition holds so long as K_g/K_o curve was shifted along the S_o axis for each change in S_w. This shift was not due to change in N, but due entirely to the change in S_w.

The ninth prediction was made to investigate the effect of connate water

saturation. With the porosity held constant at 0.25, the change in S_w caused a slight change in N. To illustrate how this change in N caused a shift of K_g/K_o curve along the S_o axis, a sample calculation was made. N is expressed in millions of stock tank barrels. When positioning the K_g/K_o curve on the S_o axis, S_o was obtained by solving Equation (7) at various values of pressure. Example: when P = 700 psia,

$$S_o = (1 - S_w)\left(1 - \frac{\Delta N}{N}\right)\frac{B}{Bi}$$

at N = 3.12 when ϕ = 0.25 and $\underline{S_w = .30}$

$$S_o = (.7)\left(1 - \frac{.41}{3.12}\right)(.865) = 0.56$$

if N = 2.9 for ϕ = 0.25 and $\underline{S_w = 3.5}$

$$S_o = (.65)\left(1 - \frac{.41}{2.9}\right)(.865) = 0.515 \text{ , a shift of } 4.5\%.$$

However, if only the change in N is reflected,

N = 2.9 for ϕ = 0.25 and $\underline{S_w = 0.30}$

$$S_o = (.7)\left(1 - \frac{.41}{2.9}\right)(.865) = .555$$

or the shift of the K_g/K_o curve along the S_o axis for a change in N of 220,000 bbls. of STO is only 0.5%. It was concluded that the small changes in N, due to changes in porosity and water saturation, were not sufficient to shift the K_g/K_o curve, but the changes in S_w did move the curve along the S_o axis a distance equal to that change. Consequently, it follows that K_g/K_o vs. S_o curve, taken as Arp's average for sandstones, is a fixed parameter except

when the value of S_w is varied.

Initial conditions for the variable connate water saturation prediction were set as follows:

P_i = 1500 psia, BPP = 900 psia, S_w = 0.25, 0.30, and 0.35, ϕ = 0.25, m = 0.1, S_{oi}* = 0.15.

The predicted results are illustrated in Figures 12.0 and 12.1.

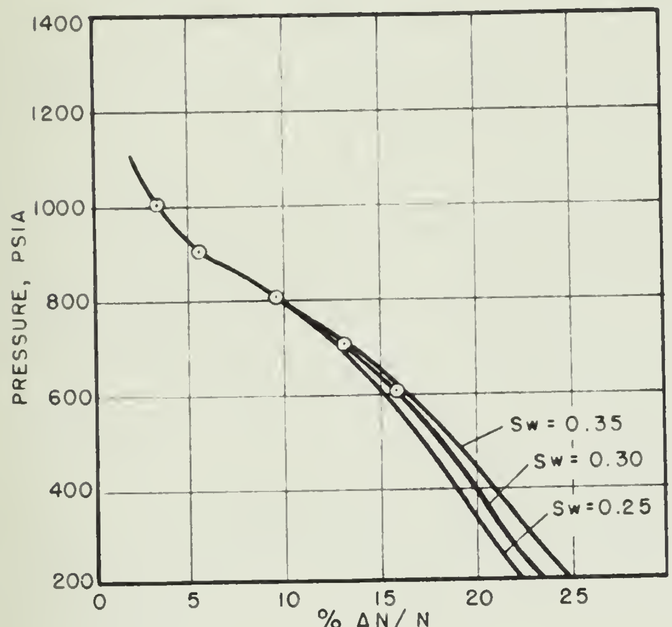


FIG. 12.0

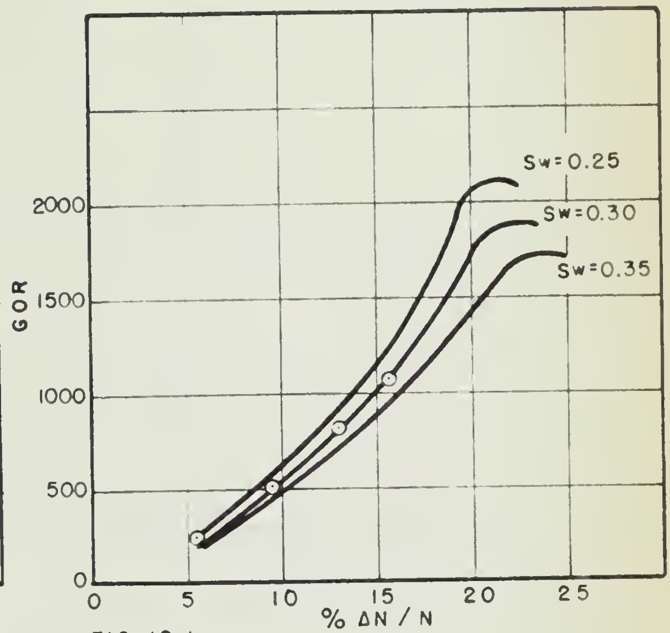


FIG. 12.1

Figures 12.0 and 12.1 illustrate the effect of the new K_g/K_o vs. S_o curves on the prediction. These curves, which were extended by Arp's average, satisfied the observed data only when S_w = 0.30.

The tenth prediction was made to investigate the effect of initial pressure. Initial conditions were set as follows:

P_i = 1500, 1400, 1300 psia, BPP = 900 psia, S_w = 0.30, ϕ = 0.25,

$m = 0.1$, $S_{oi}* = 0.15$.

The results were not of sufficient difference to merit plotting. The oil recovery at abandonment pressure of 200 psia for each P_i was:

P_i = 1500 psia, % $\Delta N/N$ = 23.57

1400 psia, % $\Delta N/N$ = 23.49

1300 psia, % $\Delta N/N$ = 23.39

It was concluded that an initial pressure from 1500 to 1300 psia will have only slight effect on recoverable oil.

The eleventh prediction illustrated the effect of changing $S_{oi}*$. The following initial conditions were set:

P_i = 1500 psia, BPP = 900 psia, $S_w = 0.30$, $\phi = 0.25$

$m = 0.1$, $S_{oi}* = 0.10$, 0.15, and 0.20.

The results indicated that, within the average limits of $S_{oi}*$ for a California solution-gas drive reservoir, S_{oi} variation has little or no effect.

It was concluded that the following parameters describe Reservoir B:

Bubble point pressure --- 900 psia

Initial reservoir pressure --- 1300 - 1500 psia

Connate water saturation --- 0.30

Porosity --- 0.25

Ratio of initial reservoir free gas volume to initial reservoir oil volume --- 0.1

Oil saturation in the gas cap --- 0.15

PVT, viscosity, and relative permeability - - - as described in Graphs A and B, respectively

Initial oil in place in Reservoir B was estimated to be 3.12 million barrels. Recoverable oil was estimated to be 730,000 barrels of stock tank oil or 23.57% of initial oil in place.

DISCUSSION OF RESULTS

The prediction results* have illustrated how quickly the reservoir engineer may bring predicted data to superimpose upon actual production data. By the fifth production, it was reasonably apparent that recoverable oil was about 23.6% of oil in place. The remaining predictions simply reaffirmed the fifth prediction by illustrating that the Case II values, selected for the variable data, gave solutions which more nearly approximated the actual production data. Such will not always be the case. However, when the reservoir engineer runs the Muskat prediction in tandem, with three different values for some doubtful input, he will very rapidly pick up a trend which will enable him to close in on the parameters which specify his reservoir.

Reservoir B was not the best possible reservoir to illustrate this problem. It is, however, typical of the situations which the reservoir engineer continuously encounters.

From the history of Reservoir B, it was obvious that gas was often

*See Table 2.0.

TABLE 2.0 Summary of Parameters Used

Prediction number	1	2	3	4	5	6	7	8	9	10
Initial Pressure	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500 1400 1300
Bubble Point Pressure	1500	1500 1200 900	1500	1500	1500 1200 900	900	900	900	900	900
Porosity	0.35	0.35	0.25	0.25	0.25	0.25	0.25	0.20 0.25 0.30	0.25	0.25
Connate Water Saturation	0.45	0.45	0.30	0.30	0.30	0.30	0.30	0.30	0.25 0.30 0.35	0.30
(m) $\frac{\text{Gas Cap Vol.}}{\text{Oil Vol.}}$	0.1	0.1	0.5	0.1 0.5 0.9	0.1	0.1 0.5 0.9	0.1	0.1	0.1	0.1
M Viscosity Ratio	1.0	1.0	1.0	1.0	1.0	1.0	0.8M 1.0M 1.5M	1.0	1.0	1.0
Case No.	1	1	2	2	2	2	2	2	2	2
Figure No.	4.0 4.1	5.0 5.1	6.0 6.1	7.0 7.1	8.0 8.1	9.0 9.1	10.0 10.1	11.0 11.1	12.0 12.1	none
% $\Delta N/N$	28.6	28.6	26.5	23.5 26.5 28.6	26.0	26.0 28.0 30.1	25 23.6 21.0	23.6	22.5 23.6 25.0	23.6

produced directly from the gas cap. Since the computer program did not account for such gas production, an estimate of the production from the gas cap was made and production data were corrected accordingly. This non-ideal situation illustrates the flexibility of the program. Water influx is not included, but may be reflected by altering the relative permeability curve. Changing the input data requires some effort, but the effort is slight when compared with that required to carry out manually the Muskat Prediction.

When the predicted results were analyzed and a set of parameters selected, it was necessary to consider whether or not the set of parameters chosen for Reservoir B was unique.

Case I illustrated that no set of parameters, employed with porosity of 0.30, 0.35 and 0.40, would satisfy observed production data. Consequently, when Case II variables were employed, it was quickly apparent that only a porosity of 0.25 gave actual observed production in barrels of stock tank oil. N was not used. After the porosity was established, only the K_g/K_o vs. S_o curve, which was positioned by $S_w = 0.30$, gave a prediction which superimposed on the observed production data. Since only this single combustion of S_w and ϕ resulted in observed production (% $\Delta N/N$ vs. Pressure) which superimposed on predicted production, the solution is a unique solution. These two variables were of primary importance because they were used to determine the observed % $\Delta N/N$. The computed % $\Delta N/N$ is a function of $(1 - S_w)$ times $1/B_o$ and does not reflect porosity at all. It follows that when an S_w satisfies both the observed and predicted % $\Delta N/N$, it is a unique value of S_w.

CONCLUSION

In this paper, the IBM-701 digital computer solution of the Muskat equation was used to predict a solution gas drive reservoir. The conclusions drawn from this analysis are as follows:

1. When using the Muskat Prediction to analyze a reservoir, one prediction is usually insufficient.
2. A match between observed and predicted data is obtained only after several calculations. During these calculations, the different parameters were varied independently and in groups.
3. Even when a match is obtained, the effect of variation of the remaining parameters must be investigated to be reasonably sure that they will not destroy the match.
4. The Pressure vs. % $\Delta N/N$ curve and the GOR vs. % $\Delta N/N$ curve could not be simultaneously matched with any set of reservoir parameters. The discrepancy between the observed and predicted GOR curves is attributed to gas production directly from the gas cap.
5. The reservoir engineer's experience and knowledge of a reservoir are a necessary part of the procedure. He must decide when the parameters best specify the reservoir. Since only the ideal reservoir will conform exactly to the predicted solution, the reservoir engineer must decide when a reasonable approximation has been made and stop his analysis at this point.

ACKNOWLEDGEMENT

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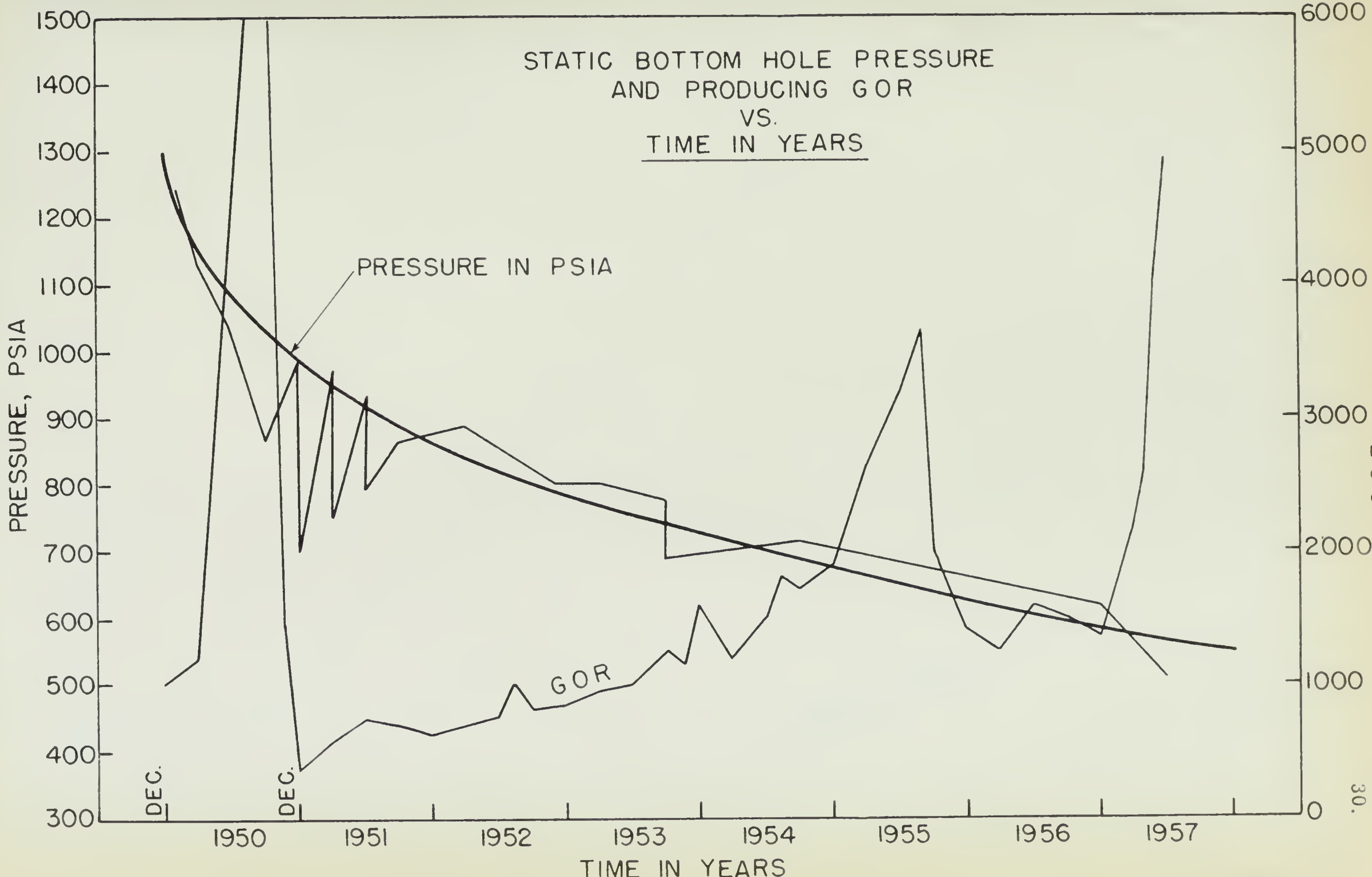
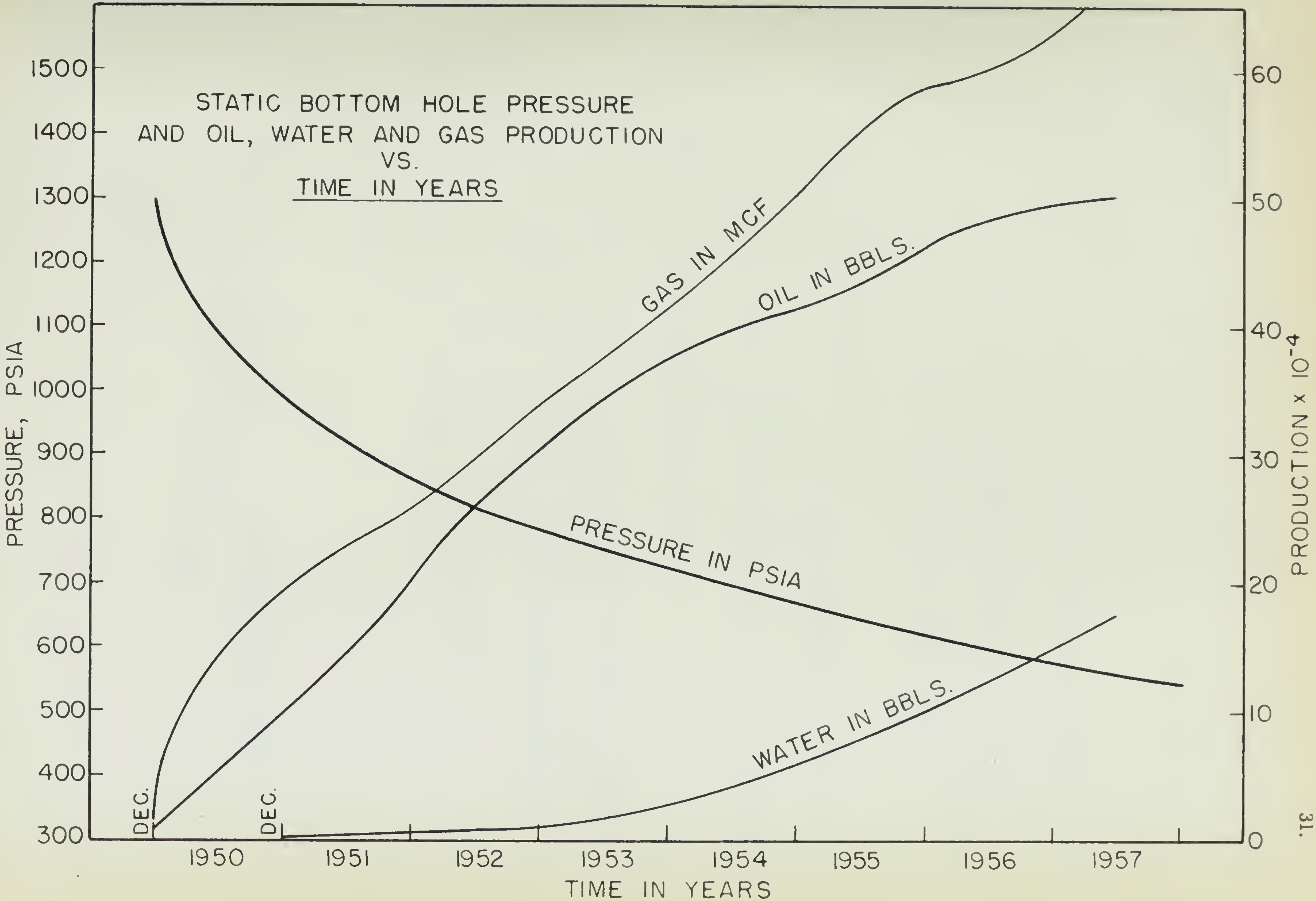

STATIC BOTTOM HOLE PRESSURE
AND PRODUCING GOR
VS.
TIME IN YEARS
PRESSURE IN PSIA
GOR
PRESSURE, PSIA
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
GOR
6000
5000
4000
3000
2000
1000
0
DEC.
DEC.
1950
1951
1952
1953
1954
1955
1956
1957
TIME IN YEARS

GRAPH C


STATIC BOTTOM HOLE PRESSURE
AND OIL, WATER AND GAS PRODUCTION
VS.
TIME IN YEARS
PRESSURE, PSIA
PRODUCTION x 10^{-4}
TIME IN YEARS
GAS IN MCF
OIL IN BBLS.
PRESSURE IN PSIA
WATER IN BBLS.
DEC.
DEC.
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60

GRAPH D

NOMENCLATURE

English

B_o - oil formation volume factor, reservoir barrels per stock tank barrel

d - differential operator

f - fraction of produced gas injected while producing below bubble-point pressure with declining pressure

G_i - cumulative gas injected, thousands of cubic feet of gas measured at standard temperature and pressure per acre-ft. of formation

G_p - cumulative gas produced, thousands of cubic feet of gas measured at standard temperature and pressure per acre-ft. of formation

i - subscript, indicates initial or starting condition of pressure or saturation

K_g - effective permeability to gas, md

K_o - effective permeability to oil, md

m - ratio of initial reservoir free gas volume to initial reservoir oil volume

n - number of pressure or saturation steps

N_p - cumulative oil produced, barrels of tank oil per acre-ft. of formation

p - pressure, psia

P_{ab} - abandonment pressure, psia

P_b - bubble-point (saturation) pressure, psia

P_{con} - constant pressure, psia

Q - see Equation 3

R - producing GOR, cubic feet of gas measured at standard temperature and pressure per stock tank barrel of oil

R_s - solution GOR (gas solubility in oil), cubic feet of gas measured at standard temperature and pressure per stock tank barrel of oil

S_g - gas saturation, fraction of pore space

S_o - oil saturation, fraction of pore space

S_{oi*} - oil saturation in the gas cap, fraction of pore space

S_w - water saturation, fraction of pore space

U - see Equation 3

X - see Equation 3

Y - see Equation 3

Greek

γ - relative gas density, cubic feet of gas measured at standard temperature and pressure per cubic foot of gas measured at reservoir temperature and pressure

Δ - difference ($p = p_1 - p_2$, $S_o = S_2 - S_1$)

μ_g - gas viscosity, cp

μ_o - oil viscosity, cp

ϕ - porosity, fraction

CALCULATION OF INITIAL OIL IN PLACE

Sand count of Figure 3: 2520 acre feet.

Case I:

$$N = \frac{7758\ (2520)\ (\phi)\ (1 - S_w)}{B_{oi}} =$$

1. B. P. = 1500, ϕ = 0.35, S_w = 0.45, B_{oi} = 1.185

$$N = \frac{(7758)\ (2520)\ (0.35)\ (0.55)}{1.185} = 3.18 \times 10^6 \text{ ST bbls}$$

2. B. P. = 900, ϕ = 0.35, S_w = 0.45, B_{oi} = 1.113

$$N = \frac{(7758)\ (2520)\ (0.35)\ (0.55)}{1.113} = 3.38 \times 10^6 \text{ ST bbls}$$

Case II:

3. B. P. = 1500, ϕ = 0.25, S_w = 0.30, B_{oi} = 1.185

$$N = \frac{(7758)\ (2520)\ (0.25)\ (0.7)}{1.185} = 2.88 \times 10^6 \text{ ST bbls}$$

4. B. P. = 900, ϕ = 0.25, S_w = 0.30, B_{oi} = 1.113

$$N = \frac{(7758)\ (2520)\ (0.25)\ (0.7)}{1.113} = 3.12 \times 10^6 \text{ ST bbls}$$

CALCULATION OF % $\Delta N/N$

Pressure psia	bbls of STO Produced	Case I %	Case II %	GOR (actual)	GOR (corrected)
1100	50,000	1.48	1.6	---	---
1000	100,000	2.96	3.2	---	---
900	175,000	5.18	5.7	600	250
800	295,000	8.75	9.5	900	500
700	410,000	12.1	13.1	1200	800
600	490,000	14.5	15.7	1670	1050

Case I: $N = 3.38$ million bbls of STO, $\phi = .35$, $S_w = .45$

Case II: $N = 3.12$ million bbls. of STO, $\phi = .25$, $S_w = .30$

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